Remarks on the Transformation of the Vertical Velocity in FLEXPART

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In `verttransform.f` the vertical velocity as read in from the ECMWF GRIB files is transformed to the co-ordinate system used in FLEXPART. This co-ordinate system is a simple form of a terrain-following co-ordinate system. If we denote its vertical co-ordinate by $\tilde{z}$,

$$\tilde{z} = z - z_0$$

where z is the regular vertical co-ordinate (metres above sea level) and z_0 is the height of the model orography. The vertical velocity in this system is

$$\tilde{w} = \dot{\tilde{z}} = d\tilde{z}/dt.$$

The ECMWF model uses a terrain following pressure co-ordinate system, called the η-system, where

$$p(\eta_k) = a_k + b_k p_0$$

with p_0 the surface pressure and k the level index (a, b are prescribed co-efficients). However, the GRIB files do not contain $\dot{\eta}$ as the vertical velocity, but (G. Wotawa, personal communciation)

$$\dot{\tilde{\eta}} := \dot{\eta}\ \partial p/\partial \eta.$$

This quantity is contained in the variable `wwh`. Presently, the transformation done is as follows:

$$\tilde{w} = \dot{\tilde{\eta}}\ (\partial z_\eta/\partial p) + \boldsymbol{v}_h \cdot \nabla_\eta \tilde{z}$$

where z_η is the height of the η levels.
However, according to Pichler (Eq. 7.200) the transformation of the vertical velocity in an arbitrary "physical" co-ordinate system with the vertical co-ordinate ϵ is

$$\dot{\epsilon} = \frac{\partial \epsilon}{\partial z}\left(w - \left.\frac{\partial z}{\partial t}\right|_\epsilon - \boldsymbol{v}_h \cdot \nabla_\epsilon\ z\right).$$

In analogy, we obtain for our problem

$$\dot{\eta} = \frac{\partial \eta}{\partial \tilde{z}}\left(\tilde{w} - \left.\frac{\partial z}{\partial t}\right|_\eta + \boldsymbol{v}_h \cdot \nabla_\eta\ \tilde{z}\right).$$

Resolving this equation for $\tilde{w}$ and using the definition of of $\dot{\tilde{\eta}}$, we obtain the desired transformation equation:

$$\tilde{w} = \dot{\tilde{\eta}}\left(\frac{\partial p}{\partial \eta}\frac{\partial \eta}{\partial z}\right)^{-1} + \left.\frac{\partial \tilde{z}}{\partial t}\right|_\eta + \boldsymbol{v}_h \cdot \nabla_\eta \tilde{z} = \dot{\tilde{\eta}}\left(\frac{\partial p}{\partial z}\right)^{-1} + \left.\frac{\partial \tilde{z}}{\partial t}\right|_\eta + \boldsymbol{v}_h \cdot \nabla_\eta \tilde{z}.$$

We can see that one term is missing in the original transformation, namely the change of the height of a material fluid particle in time due to the vertical movement of the η surface, being caused by temperature and/or humidity changes in the air column below this particle, or changes in the mass of this column. A back-of-the-envelope calculation gives a possible magnitude of this term of 100 m per day in the middle troposphere, a non-negligible quantity.